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Simplified Radiation Analysis Using Modularized Enclosures

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Nomenclature

A_i	= area of node i
F_{i-j}	= over-all radiant interchange factor from i to j
i	= ith node of enclosure or subenclosure
j	= jth subenclosure
m,m'	= *MESS* node located in subenclosure j and adjacent subenclosures, respectively
n	= nth subenclosure
N	= number of nodes in thermal network enclosure or subenclosure
P_i	= number of primary radiators for node i
Q_i	= total heat load for node i
r,r'	= *MESS* node located in subenclosures n and j, respectively
R_n,R_j	= number of *MESS* node interfaces in subenclosures n and j, respectively
T	= absolute temperature
T_i,T_i'	= exact and approximate temperature of ith node
γ	= primary radiation fraction, radiators of a node left intact divided by the sum of the node radiators
σ	= Stefan-Boltzmann constant

Subscripts

ERN	= effective radiation steady-state node
h,k,l	= enclosure, subenclosure nodes
MESS	= multiple enclosure simplification shield

THE Radiation Condenser (RC) computer program[1] has been generalized to include a new technique of enclosure radiative analysis simplification referred to as the Multiple Enclosure Simplification Shield (*MESS*) technique.[2] The generalized RC computer program presented in this Note makes application of the *ERN/MESS* technique practical in the MITAS[3] thermal analyzer and provides a single tool for radiation analysis optimization. The program is presently being used on the NASA Skylab and Viking projects where computer run times have been reduced as much as 80%.

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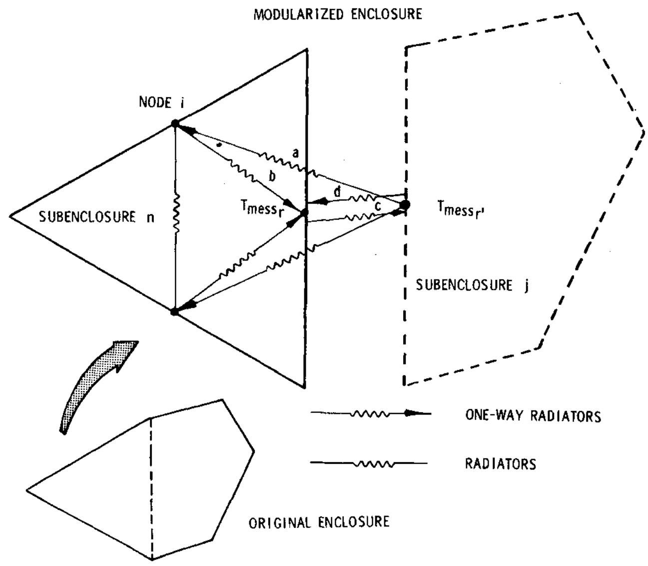

Fig. 1 *MESS* technique one-way radiators.

The *ERN* technique is used to reduce the number of radiative couplings (radiators) required to thermally model enclosures. *MESS* enables a complex enclosure to be modularized into discrete subenclosures by assigning imaginary interface shield nodes. The smaller enclosures can be analyzed efficiently with existing computers. Additionally, each subenclosure can be analyzed independently of the others which may result in more efficient use of manpower.

ERN/MESS Technique

The *ERN* and *MESS* techniques are independent and may be discussed separately. Consider an N node radiative enclosure that forms a section of a complex thermal network. The temperature of node i is a function of radiation, conduction, convection, and applied heat loads. Also assume that the contribution to the temperature of node i from the conduction and convection terms are included in the heat load Q_i. The steady-state temperature of node i is

$$T_i = \left[\left(\sum_{k=1}^{N} \sigma A_i F_{i-k} T_k^4 + Q_i\right) \Big/ \sum_{k=1}^{N} \sigma A_i F_{i-k}\right]^{1/4} \quad (1)$$

Application of the *ERN* technique requires that the radiators of node i be divided into P_i primary and $N - P_i$ secondary terms. Radiators for node i can then be written:

$$\sum_{k=1}^{N} \sigma A_i F_{i-k} T_k^4 = \sum_{k=1}^{Pi} \sigma A_i F_{i-k} T_k^4 + \sum_{l=P_{i+1}}^{N} \sigma A_i F_{i-l} T_l^4 \quad (2)$$

The number of radiators is reduced by replacing the secondary radiator summation with a single term coupled to an *ERN*. The temperature is calculated by the thermal analyzer program as a radiator-weighted steady-state temperature using secondary radiator sums.

$$T_{ERN} = \left[\sum_{i=1}^{N} \sum_{l=P_{i+1}}^{N} \sigma A_i F_{i-l} T_i^4 \Big/ \sum_{i=1}^{N} \sum_{l=P_{i+1}}^{N} \sigma A_i F_{i-l}\right]^{1/4} \quad (3)$$

Application of the *MESS* technique enables a radiative enclosure to be divided into an arbitrary number of subenclosures. *MESS* node pairs are defined by the analyst at the interface between subenclosures as two planer surfaces with the property of absorbing and emitting all energy incident upon them (black surfaces). Consider an N node subenclosure, n, shown in Fig. 1 where subscripts r and r' refer to the *MESS* node pair of the nth and jth subenclosures, respectively. Temperatures in n are affected by $T_{MESSr'}$ that represents the average thermal effect of j nodes on n. The primary radiators of Eq. (2) include *MESS* radiators. For n, with R_n interface *MESS* nodes, the primary radiative couplings for node i are

$$\sum_{k}^{Pi} \sigma A_i F_{i-k} T_k^4 = \sum_{r=1}^{R_n} \sigma A_i F_{i-r} T_{MESSr'}^4 + \sum_{l=R_{n+1}}^{P_i} \sigma A_i F_{i-l} T_l^4 \qquad (4)$$

Using an energy balance on r' gives

$$T_{MESSr}' = \left[\left(\sum_{\substack{m=1\\ m'\neq r'}}^{Rj} \sigma A_{r'} F_{r'-m}\, T_{MESSm'}^4 + \sum_{k=R_{j+1}}^{P_{r'}} \sigma A_{r'} F_{r'-k} T_k^4 + \sigma A_{r'} F_{r'-r} T_{MESSr}^4\right) \Bigg/ \left(\sum_{\substack{m=1\\ m\neq r'}}^{Rj} \sigma A_{r'} F_{r'-m} + \sum_{k=R_{j+1}}^{P_{r'}} \sigma A_{r'} F_{r'-k} + \sigma A_{r'} F_{r'-r}\right)\right]^{1/4} \qquad (5)$$

where T_{MESSr} is in n and $F_{r'-r}$ has the value of the radiant interchange factor of r' to itself and comes from the radiant interchange analysis of j. $F_{r'-r}$ represents the reflections between n and j due to nonblack subenclosure surfaces and is obtained from the radiation interchange matrix for each subenclosure. This matrix is calculated using the techniques of Hottel or Gebhart.[4]

The approximate temperature of the ith node is obtained from Eq. (1) using Eqs. (3) and (4) as:

$$T_i' = \left\{\left[\sum_{r=1}^{Rn} \sigma A_i F_{i-r} T_{MESSr'}^4 + \sum_{l=R_{n+1}}^{P_i} \sigma A_i F_{i-l} T_l^4 + \left(\sum_{h=P_{i+1}}^{N} \sigma A_i F_{i-h}\right) T_{ERN}^4 + Q_i\right] \Bigg/ \left(\sum_{r=1}^{Rn} \sigma A_i F_{i-r} + \sum_{l=R_{n+1}}^{P_i} \sigma A_i F_{i-l} + \sum_{h=P_{i+1}}^{N} \sigma A_i F_{i-h}\right)\right\}^{1/4} \qquad (6)$$

The error in T_i' is a complex function of the percentage of *ERN* secondary radiators, temperature band of the subenclosure nodes and the number of subenclosure divisions. A variety of problems have been studied, and the error has been found to be negligible. Empirical guidelines have been determined.

Application

ERN and *MESS* technique equations can be included in thermal analyzer programs directly by using subroutines. A more convenient method is to represent the equations using the standard resistance-capacitance network couplings. *ERN* radiators are coupled directly to subenclosure nodes. Incorporation of the *MESS* technique requires the use of one-way couplings commonly used to represent fluid flow. A one-way coupling (conduction or radiation) between two nodes enters into the temperature calculation of only one node. Each radiator from *MESS* node r to subenclosure node i is replaced by two one-way radiators with values equal to that of the original radiator. These one-way radiator pairs are shown in Fig. 1 (radiators a and b). One-way radiators c and d couple the primary and secondary *MESS* node pair to account for reflections between a subenclosure pair. The values of radiators c and d are determined from the radiation interchange matrix analysis of n and j, respectively.

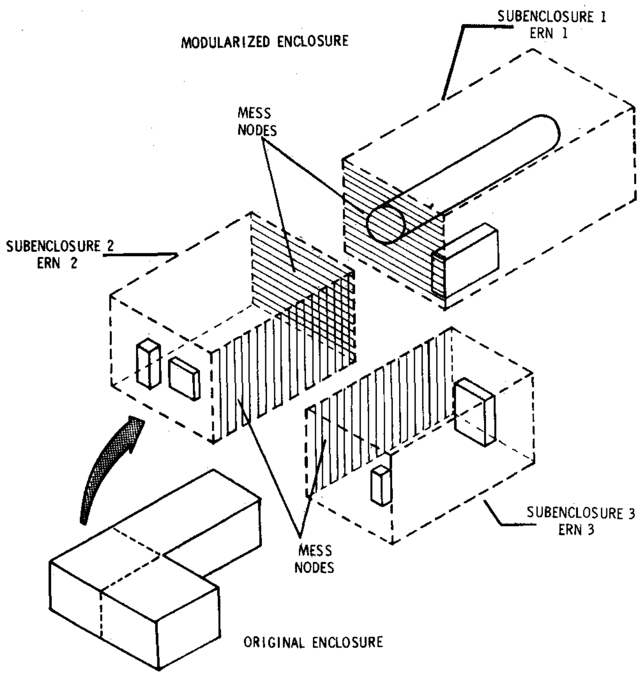

Fig. 2 Apollo Telescope Mount HAO experiment optics housing sample problem.

ERN/MESS Computer Program

The RC program generates *ERN* radiators and *MESS* one-way radiators from the subenclosure radiant interchange matrix. The program technique is simple and can be adapted to any radiant interchange computer program.

Generation of *ERN* couplings requires that the analyst assign the primary radiation fraction γ as an RC program input. Node radiators are arranged by the program in decreasing order of the radiator $(A_i F_{i-j})$ value. The program operates on each node selecting the largest remaining radiator values. This procedure is terminated when the sum of the selected radiators divided by the sum of the node's radiators is greater than the fraction γ. Reverse direction radiators are flagged, and the remaining radiators for each node are summed and coupled to the *ERN*. Since the error is a function of the enclosure temperature gradients, the *ERN* approximation can be improved if nodes that deviate significantly in temperature from the average subenclosure temperature are not coupled to the *ERN*. These analyst-defined nodes are referred to as special nodes in the RC program.

Generation of *MESS* one-way radiators from the subenclosure radiant interchange matrix requires that the analyst specify the interface *MESS* node pair. Subenclosure *MESS* nodes are considered as special nodes by the RC program. As node radiators are generated, *MESS* nodes are flagged, and appropriate one-way radiators are generated for use in the thermal analyzer program. For the MITAS thermal analyzer radiators, the unaffected node in the temperature calculation is flagged with a minus sign. Radiator one-way couplings between nonpaired *MESS* nodes of a subenclosure are also generated. The value of each radiator of the one-way pair is equal to the radiant interchange factor between the subenclosure's two *MESS* nodes.

The *ERN/MESS* technique has been verified for a complex geometry typical of spacecraft enclosures. The optics housing on the High Altitude Observatory solar telescope, which is mounted on the Skylab Apollo Telescope Mount, is shown in Fig. 2. The 30 node thermal model includes transient boundary conditions. The original enclosure is shown along with the modularized subenclosure that contains *ERN* and *MESS* steady-state nodes. Subenclosure configuration factors and radiation networks were generated independently; the RC program was applied using a γ of 0.7, and the integrated network was input to the MITAS program. Steady-state and transient results indicate negligible error when the *ERN/MESS* enclosure is compared to complete enclosure analysis results. Boundary temperatures and heat fluxes imposed on the model produced internal component temperature variations of greater than 100°F. Even with these severe temperature gradients, the largest temperature deviation between the original and modularized subenclosure was 14°F at a temperature level of 441°F.

The location of *MESS* node pairs inside an enclosure is influenced by the following: number of subenclosure surfaces, geometric considerations, expected thermal gradients and the number of analysts that can work on the enclosure. Optimum reduction in configuration factors and radiators occurs in large enclosures (over 20 nodes) if there are an equal number of nodes in each subenclosure. For enclosures divided into two subenclosures, each having an equal number of surfaces, up to 50% reduction in the number of configuration factors and radiators can be expected.

The number of radiators for each subenclosure is optimized using the *ERN* technique. For a constant γ, experience has shown that the greatest percentage reduction in radiators is for large subenclosures (greater than 75 nodes) with significant shadowing and low emittance surfaces. The smallest radiator reduction is for subenclosures with a low number of nodes (less than 30) with a symmetrical geometry. A γ of 0.7 has been found to be a good compromise resulting in significant reduction in couplings with minimal error.

The *ERN/MESS* technique reduces the number of configuration factors and radiators necessary for enclosure radiation analysis and extends the analysis to enclosure arbitrary complexity by creating artificial boundary surfaces within complex enclosures. Computerized *ERN/MESS* application using the RC program is general and may be adapted to other radiator/thermal analyzer programs, resulting in considerable analyst and computer savings.

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